Title: Beginning Modeling for Linear and Quadratic Data

Link to Outcomes:

- **Techniques** Students will gather and analyze data from both real-world situations and student constructed models.
- **Communication** Students will present collected data in table form, translate it into graphs, then produce linear or quadratic equations representing lines of best fit.
- **Problem Solving** Students will demonstrate their ability to make decisions based on observed data, then extrapolate further data from these observations.
- **Measurements** Students will construct triangles of given dimensions, then estimate the areas of the figures.
- **Technology** Students will use the TI-82 graphics calculator to graph data and to identify the best linear or quadratic function rule for a set of data.
- **Reasoning** Students will determine whether each set of data is linear or quadratic.
- **Algebra** Students will develop a linear or quadratic equation model for their data

Brief Overview:

This lesson will deal with modeling using three different sets of data. For two of the sets, students will collect the data themselves, make tables, then graph and find equations on the calculator. It can be done in pairs or groups. For the third set, students will use given data to graph, find equations, and then extrapolate their results.

Grade/Level:

Grades 8–10 Algebra I, Algebra II

Duration/Length:

This lesson will take 2 to 3 periods (45 min.).

Prerequisite Knowledge:

Students should be familiar with the concept of area, recognizing linear and quadratic graphs, and using the TI-82 calculator.

Objective:

- To gather and present data in table form.
- To use the TI-82 to store data, graph, and find equations of best fit.
- To interpret and extrapolate data.

Materials/Resources/Printed Materials:

- Graph paper
- Ruler
- TI-82 or TI-85 graphics calculator
- Worksheets

Development/Procedures:

Students are divided into pairs or groups. They work together to complete the tables. Teacher direction may be necessary for the calculator steps in some classes. The students will graph each set of values, make a scatter plot, then find the equation of best fit.
The teacher notes provide specific suggestions for each activity.

Evaluation:

Assessment will be based on group performance in data collection and the processing of that data.

Extension/Follow Up:

Students visit the media center and use the almanac to find lists of data. They will then use this data to predict future trends.

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Varying the Height of a Triangle vs. Area

Directions:
1. You will need graph paper and a ruler.
2. For each height in the table below, use the graph paper to draw a triangle with a base of 4 units.
3. Find an estimate of the area of each triangle by counting the number of squares within each triangle and record your results below.

Height	Estimate of Area
2	
3	
4	
5	
6	
7	
8	
9	

Height	Estimate of Area
10	
11	
12	
13	
14	
15	
16	
17	

4. Enter the values for height into the calculator for L_1 and area into L_2. Make a scatter plot of the values. Sketch the results below.

5. Would this be a linear or quadratic function? Find the equation of best fit by trying both linear and quadratic regression equations and graphing them over the scatter plot. Write and sketch the graph of this best fit equation below.

Equation: ___________________________

Extensions:

1. What would be the area of a triangle with base 4 and height 20 units? height 30 units?

2. Would this same equation give accurate areas if the base had not been 4 units? Give examples to prove your answer.

Areas of Equilateral Triangles

Directions:
1. You will need graph paper and a ruler.
2. Using each value below, sketch an equilateral triangle.
3. Find an estimate of the area of each triangle by counting the number of squares within each triangle and record your results below.

Length of Side	Estimate of Area
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	

4. Enter the values for length of side into the calculator for L_1 and area into L_2. Make a scatter plot of the values. Sketch the results below.

5. Would this be a linear or quadratic function? Find the equation of best fit by trying both linear and quadratic regression equations and graphing them over the scatter plot. Write and sketch the graph of this best fit equation below.

Equation: ________________________________

Extension:

1. Use the trace feature to find an estimate of the area of an equilateral triangle with side 24 units. Use your equation to check this estimate.
2. (Advanced classes) Using s as the length of a side, develop a formula for the area of any equilateral triangle. Use this formula to compare your estimates with the actual areas.

Teachers Notes and Answer Key

Varying the Height vs. Area

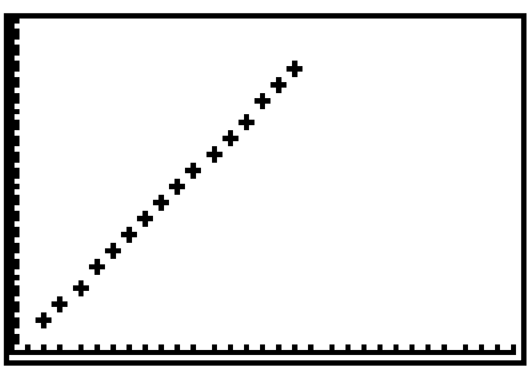
1. Students could work in pairs or in groups.
2. Have each student do part of the activity then exchange with a partner to check results.
3. Students then combine results to complete the table.
4. Answers will vary but should be close to the key below which assumes a base of four.

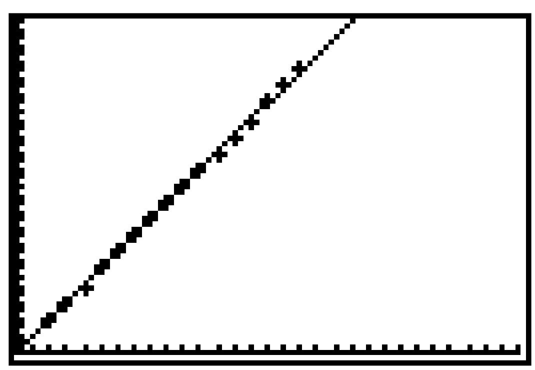
Height	Estimate of Area
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

Height	Estimate of Area
10	20
11	22
12	24
13	26
14	28
15	30
16	32
17	34

5. Steps #4 and #5 could be done as a class if the students are not proficient in using the graphing calculator. More advanced classes could finish as pairs or in groups.

Equation: $y = 2x$ Height 20 units = Area 40 units
Height 30 units = Area 60 units
Equation would not work for different bases since A = 1/2 bh



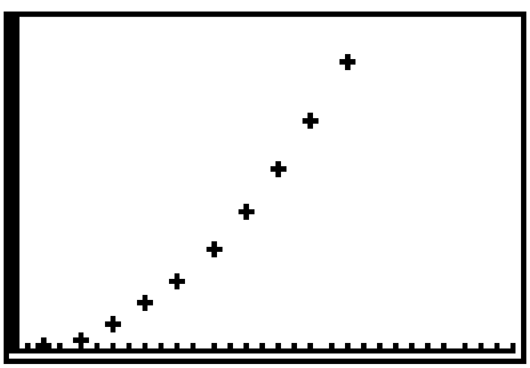


Teachers Notes and Answer Key

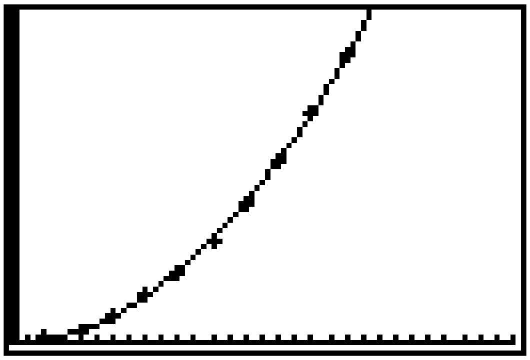
Areas of Equilateral Triangles

1. Answers will vary but should be close to the values below.
2. Students could work individually or in pairs or groups.

Length of Side	Estimate of Area
2	1.7
4	6.9
6	15.6
8	27.7
10	43.3
12	62.4
14	84.9
16	110.9
18	140.3
20	173.2



QuadReg
$y=ax^2+bx+c$
$a=.4324810606$
$b=.0142045455$
$c=^{-}.0683333333$



3. Side 24 units → Area approximately 249.4
4. Geometric formula for area of equilateral triangle: A = side squared times square root of three all divided by 4. This can be derived by drawing an altitude and using the 30-60-90 relationships.

THE OLYMPIC POLE VAULT

The following data describes the Olympic Pole Vault heights in inches for all the modern Olympics.

Years	Inches
1896	130"
1900	130"
1904	137.5"
1908	146"
1912	155.5"
1920	161"
1924	156.5"
1928	165.25"
1932	169.75"
1936	171.25"
1948	169.25"

Years	Inches
1952	179"
1956	180.5"
1960	185"
1964	200.75"
1968	212.5"
1972	217.5"
1976	217.5"
1980	227.5"
1984	226.25"
1988	237.25"
1992	228.25"

Enter the above data into your TI-82 calculator and then:

1. Find the linear equation that “fits” your data.
2. Find the quadratic equation that “fits” your data.
3. Graph the data using STAT PLOT.
4. Graph the linear equation and the linear equation at the same time as you graph your STAT PLOT graph.
5. Use #4 to decide which equation "fits" best.

EXTENSIONS

A. Instead of using years in LIST 1, substitute 96 for 1896, 100 for 1900, 104 for 1904, etc. and repeat #1 through #5 above.

B. Instead of using years in LIST 1, substitute 1 for 1896, 2 for 1900, 3 for 1904, etc. and repeat #1 through #5 above.

C. Which LIST 1 data do you feel is the “best”? Justify your answer by writing a short explanation.

D. Finally, use each linear equation and each quadratic equation obtained and predict the winning height for the 1996 Olympics (you should have 6 different predictions).

TEACHER NOTES FOR MAKING SCATTER DRAWINGS AND LINEAR AND QUADRATIC EQUATIONS

To make a scatter plot:

1. Students must enter data by using their {STAT} button, EDIT, {1} and by entering the first list into L_1 and entering the second list into L_2.

2. Students make a scatter plot of their original data by pushing {2ND}, STAT PLOT, {1}, {ENTER} and checking to make sure that On, Type, L_1 list, L_2 list and Mark are properly highlighted before pushing {ZOOM} {9}.

To graph line of best fit:

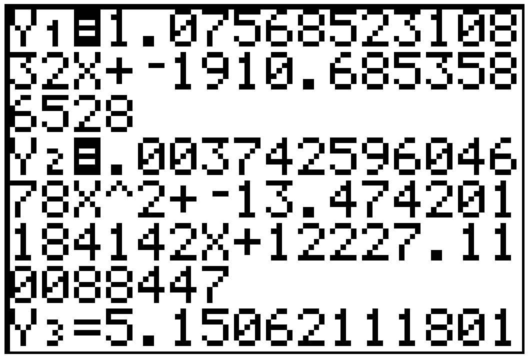
3. Students graph their linear or quadratic equations by first clearing all equations in {Y=}.

4. Students obtain their linear equation by using the {STAT} button, {CALC}, {5}, and {ENTER}. To obtain their quadratic equation use {STAT}, {CALC}, {6}, and {ENTER}. To graph the equation push {Y=}, {VARS}, {5}, toggle right to EQ, then {7}. The equation is now in Y1. Push {GRAPH}. Students may have to adjust {WINDOW} when using the {TRACE} button.

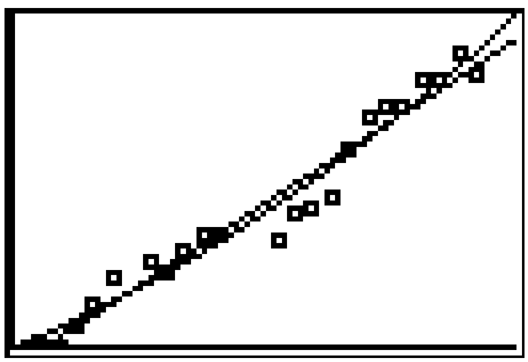
TEACHER KEY FOR <u>THE OLYMPIC POLE VAULT</u>

Since the OLYMPIC POLE VAULT sheet need not be a true worksheet, the teacher may choose to assign the problems one at a time and walk their students through the problems.

The answers for the linear and quadratic equations when the first list is in years; i.e., 1896, 1900, 1904,…, 1992 and the second list is “inches.”

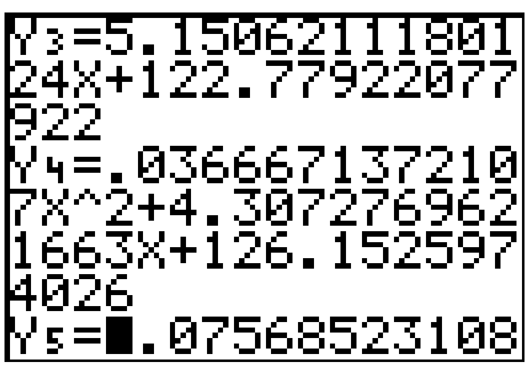


The graph of the STAT PLOT and the linear and quadratic equations above: the window is [1890, 2000] by [130,250].

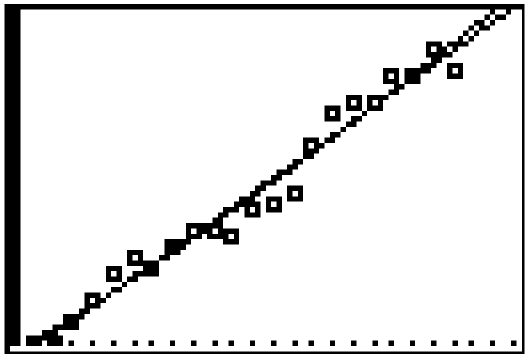


There are various answers as to which equation is the “best” fit.

The answers for the linear and quadratic equations when the first list is 1, 2, 3,…,22 and the second list is “inches.”

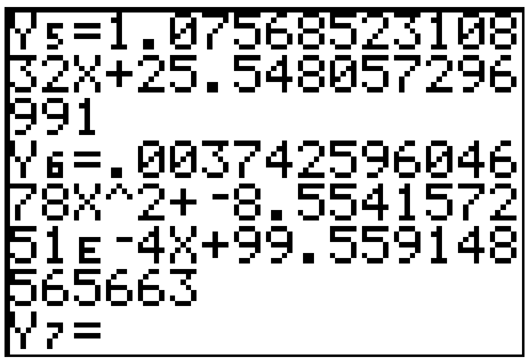


The graph of the STAT PLOT and the linear and quadratic equations above; the window is [0, 25] by [130,250].

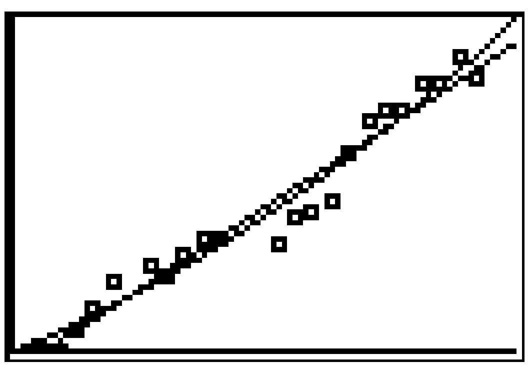


There are various answers as to which equation is the “best” fit.

The answers for the linear and quadratic equations when the first list is 96, 100, 104,…, 192 and the second list is “inches.”



The graph of the STAT PLOT and the linear and quadratic equations above: the window is [90,200] by [130,250].



There are various answers as to which equation is the “best” fit.

For problem D) the predictions are:

x= 1996	x=23	x=196
linear: 236.38"	linear: 241.24"	linear: 236.38"
quadratic: 243"	quadratic: 244.61"	quadratic: 243.16"